量测随机延迟下带厚尾噪声的鲁棒 Student’s t 随机容积卡尔曼滤波器

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摘　要: 针对量测随机延迟下带厚尾过程噪声和量测噪声的非线性状态估计问题,本文通过充分考虑量测一步随机延迟特性及过程噪声和量测噪声的“厚尾”特性,推导了一种新的鲁棒 Student’s t 滤波器框架,并采用随机 Student’s t-球面相径容积规则近似计算 Student’s t 权值积分,从而设计了一种新的鲁棒 Student’s t 随机容积滤波器.首先,采用一组服从伯努利分布的随机序列来描述系统中可能存在的量测一步随机延迟现象,并采用 Student’s t 分布刻画过程噪声和量测噪声中存在的“厚尾”特性;其次,从理论上证明了当自由度参数趋于无穷以及随机延迟概率为零时,该鲁棒 Student’s t 滤波器就自动地降为标准的非线性高斯近似滤波器;最后,采用随机 Student’s t-球面相径容积规则给出了一种新的鲁棒 Student’s t 随机容积滤波器,并通过协同转弯模型验证了该滤波器的有效性和优越性.

关键词: 随机延迟; 厚尾噪声; Student’s t 权值积分; 伯努利分布; 矩匹配方法; 非线性估计
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Robust Student’s t-Based Stochastic Cubature Kalman Filter with Heavy-Tailed Noises and Randomly Delayed Measurements

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Abstract: Aiming at the nonlinear state estimation problem with heavy-tailed process measurement noise and randomly delayed measurements, this paper deduces a new robust student’s t filter framework by taking into one-step random delay characteristics and heavy-tailed characteristics of process noise and measurement noise account fully. Meanwhile, this paper proposes a new robust Student’s t-based stochastic cubature filter (RSTCF) by approximately calculating the Student’s t weight integral using stochastic Student’s t-spherical radial cubature rule. Firstly, this paper uses a set of stochastic sequences obeying the Bernoulli distribution to describe the possible one-step random delay phenomenon in the system, what’s more, this paper also uses the student’s t distribution to characterize the heavy-tailed characteristics of the process noise and measurement noise. Secondly, it’s theoretically proved that the robust Student’s t filter automatically will be reduced to a standard nonlinear Gaussian approximation filter when the degrees of freedom in the posterior probability density function of the state and measurement noise are increasing continuously and randomly delay probability is equal to zero. Finally, this paper presentes a new robust Student’s t-based stochastic cubature filter by use of stochastic Student’s t-spherical radial cubature rule. The effectiveness and superiority of the filter are verified by the coordinated turn maneuvers model.

Key words: randomly delay; heavy-tailed noises; student’s t weighted integrals; Bernoulli distribution; moment matching method; nonlinear estimation

1 引言

离散时间非线性动态系统的状态估计已经被广泛地应用于目标跟踪与定位[1]、信号处理[2]、组合导航[3]、自动控制[4]等领域. 一般地,对于非线性系统而言,在贝叶斯框架下,基于量测信息的状态后验概率密

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度函数(Probability Density Function,PDF)的递推最优解通常是难以获得的. 因此,可以通过合理的近似获得非线性系统的状态的次优估计. 高斯近似是一种被广泛使用的近似方法,目前基于不同的积分规则,已经提出了许多次优的高斯近似滤波器[5~8].

上述文献中介绍的非线性高斯近似滤波器需要假设量测数据能够实时到达数据处理中心,且系统噪声和过程噪声均服从高斯分布,但是,在实际工程应用中,由于通信信道带宽的限制导致量测抵达数据处理中心时不可避免地存在随机延迟现象[9,10],从而导致量测数据无法及时得到更新,并且,由于不可靠传感器的使用导致采集到的量测数据存在一定的野值,野值数据的存在使得高斯分布的尾部变大,具有明显的厚尾特征[11]. 同时,在实际目标跟踪过程中,由于目标的剧烈机动,使得用于描述目标运动的非线性动态系统的过程噪声同样具有明显的厚尾特征[12]. 由于量测随机延迟现象及厚尾过程噪声和量测噪声的存在导致了上述非线性高斯近似滤波器的估计性能变差甚至发散. 一方面,为了解决量测随机延迟条件下的状态估计问题,目前已经涌现出了一系列改进的高斯近似滤波器,并取得了较好的状态估计性能[9,13~15]. 另一方面,针对厚尾过程噪声和量测噪声条件下的状态估计问题,目前所采取的解决方法主要有蒙特卡罗法、高斯和滤波器、基于 Huber 方法的鲁棒卡尔曼滤波器[16,17]、基于最大熵准则的最大熵卡尔曼滤波器[18~20]以及鲁棒 Student's t 滤波器[12,21~28]等.

上述文献所设计的非线性状态估计滤波器在量测随机延迟及厚尾过程噪声和量测噪声条件下,将不可避免的面临着状态估计发散的风险和计算效率低下的问题. 针对这种情形,本文通过充分考虑量测一步随机延迟特性以及过程噪声与量测噪声的厚尾特性,推导了一种适用于量测随机延迟和带厚尾过程噪声和量测噪声条件下状态估计的鲁棒 Student's t 滤波框架,并基于随机 Student's t-球面相径容积规则,提出了一种新的鲁棒 Student's t 随机容积滤波器,最后采用目标跟踪仿真验证本文算法的优越性.

2　问题描述

考虑如下非线性离散随机系统

$$\boldsymbol{x}_k = f_{k-1}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k-1} \tag{1}$$

一步随机延迟量测模型为

$$\boldsymbol{z}_k = h_k(\boldsymbol{x}_k) + \boldsymbol{v}_k \tag{2}$$

$$\boldsymbol{y}_k = (1-\gamma_k)\boldsymbol{z}_k + \gamma_k \boldsymbol{z}_{k-1}, k>1; \boldsymbol{y}_1 = \boldsymbol{z}_1 \tag{3}$$

式中,$k \in \{1,2,\cdots\}$,$\boldsymbol{x}_k \in \mathbb{R}^n$ 是状态变量,$\boldsymbol{z}_k \in \mathbb{R}^m$ 是理想的量测向量,$\boldsymbol{y}_k \in \mathbb{R}^m$ 是实际的量测向量,$h_k(\cdot)$ 为非线性量测函数,过程噪声 $\boldsymbol{w}_k \in \mathbb{R}^n$ 且服从 t 分布 $\mathrm{St}(\boldsymbol{w}_k;\boldsymbol{0},\boldsymbol{Q}_k,v_1)$,$\boldsymbol{v}_k \in \mathbb{R}^m$ 且服从 t 分布 $\mathrm{St}(\boldsymbol{v}_k;\boldsymbol{0},\boldsymbol{R}_k,v_2)$. $\{\gamma_k \in \mathbb{R}, k>1\}$ 为已知的服从 Bernoulli 分布的随机变量,取值范围在 0 和 1 之间且满足

$$\begin{cases} p(\gamma_k = 1) = E[\gamma_k] = \lambda \\ p(\gamma_k = 0) = 1 - E[\gamma_k] = 1-\lambda \\ E[(\gamma_k - \lambda)^2] = (1-\lambda)\lambda \end{cases} \tag{4}$$

初始状态 $\boldsymbol{x}_0$ 与过程噪声 $\{\boldsymbol{w}_k, k \ge 0\}$,$\{\boldsymbol{v}_k, k \ge 1\}$ 以及 $\{\gamma_k, k \ge 2\}$ 相互独立并且服从 t 分布 $\mathrm{St}(\boldsymbol{x}_0;\hat{\boldsymbol{x}}_{0|0},\boldsymbol{P}_{0|0},v_3)$. $\boldsymbol{P}^{xy}_{k+1|k}$ 为量测随机延迟概率,一般假设已知.

3　量测随机延迟下带厚尾过程噪声和量测噪声的鲁棒 Student's t 滤波器

在给出量测随机延迟下带厚尾过程噪声和量测噪声的鲁棒 Student's t 滤波器的一般框架解之前,首先给出两个假设:

假设 1　状态变量 $\boldsymbol{x}_k, \boldsymbol{x}_{k+1}$ 和随机延迟量测 $\boldsymbol{y}_{k+1}$ 在延迟量测集合 Y_k 的条件下的联合分布服从 Student's t 分布,即

$$p(\boldsymbol{x}_k, \boldsymbol{x}_{k+1}, \boldsymbol{y}_{k+1} | Y_k) = \mathrm{St}\left(\begin{bmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \\ \boldsymbol{y}_{k+1} \end{bmatrix}; \begin{bmatrix} \hat{\boldsymbol{x}}_{k|k} \\ \hat{\boldsymbol{x}}_{k+1|k} \\ \hat{\boldsymbol{y}}_{k+1|k} \end{bmatrix}, \begin{bmatrix} \boldsymbol{P}_{k|k} & \boldsymbol{P}_{k+1,k|k} & \boldsymbol{P}^{xy}_{k,k+1|k} \\ \boldsymbol{P}^{\mathrm{T}}_{k+1,k|k} & \boldsymbol{P}_{k+1|k} & \boldsymbol{P}^{xy}_{k+1|k} \\ (\boldsymbol{P}^{xy}_{k,k+1|k})^{\mathrm{T}} & (\boldsymbol{P}^{xy}_{k+1|k})^{\mathrm{T}} & \boldsymbol{P}^{yy}_{k+1|k} \end{bmatrix}, v_3\right) \tag{5}$$

其中,状态估计量 $\boldsymbol{x}_{k|k}$ 及 $\boldsymbol{P}_{k|k}$ 分别是概率密度函数 $p(\boldsymbol{x}_k | Y_k)$ 的均值和尺度矩阵;状态预测量 $\boldsymbol{x}_{k+1|k}$ 及 $\boldsymbol{P}_{k+1|k}$ 分别是概率密度函数 $p(\boldsymbol{x}_{k+1} | Y_{k+1})$ 的均值和尺度矩阵;量测预测量 $\hat{\boldsymbol{y}}_{k+1|k}$ 及 $\boldsymbol{P}^{yy}_{k+1|k}$ 分别是概率密度函数 $p(\boldsymbol{y}_{k+1} | Y_k)$ 的均值和尺度矩阵;$\boldsymbol{P}_{k+1,k|k}$ 为状态 $\boldsymbol{x}_k$ 和状态 $\boldsymbol{x}_{k+1}$ 的互尺度矩阵;$\boldsymbol{P}^{xy}_{k,k+1|k}$ 为状态 $\boldsymbol{x}_k$ 和量测 $\boldsymbol{y}_{k+1}$ 的互尺度矩阵;$\boldsymbol{P}^{xy}_{k+1|k}$ 为状态 $\boldsymbol{x}_{k+1}$ 和量测 $\boldsymbol{y}_{k+1}$ 的互尺度矩阵. $\boldsymbol{x}_{k|k}$、$\boldsymbol{x}_{k+1|k}$、$\hat{\boldsymbol{y}}_{k+1|k}$、$\boldsymbol{P}_{k|k}$、$\boldsymbol{P}_{k+1|k}$、$\boldsymbol{P}^{yy}_{k+1|k}$、$\boldsymbol{P}^{xy}_{k,k+1|k}$、$\boldsymbol{P}^{xy}_{k+1|k}$ 可分别通过下式获得

$$\begin{cases} \hat{\boldsymbol{x}}_{k|k} = E[\boldsymbol{x}_k | Y_k]\, \boldsymbol{P}_{k|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{x}}_{k|k}\tilde{\boldsymbol{x}}^{\mathrm{T}}_{k|k} | Y_k] \\ \hat{\boldsymbol{x}}_{k+1|k} = E[\boldsymbol{x}_{k+1} | Y_k]\, \boldsymbol{P}_{k+1|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{x}}^{\mathrm{T}}_{k+1|k} | Y_k] \\ \hat{\boldsymbol{y}}_{k+1|k} = E[\boldsymbol{y}_{k+1} | Y_k]\, \boldsymbol{P}^{yy}_{k+1|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{y}}_{k+1|k}\tilde{\boldsymbol{y}}^{\mathrm{T}}_{k+1|k} | Y_k] \\ \boldsymbol{P}_{k+1,k|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{x}}^{\mathrm{T}}_{k|k} | Y_k] \\ \boldsymbol{P}^{xy}_{k,k+1|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{x}}_{k|k}\tilde{\boldsymbol{y}}^{\mathrm{T}}_{k+1|k} | Y_k] \\ \boldsymbol{P}^{xy}_{k+1|k} = \dfrac{v_3 - 2}{v_3} E[\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{y}}^{\mathrm{T}}_{k+1|k} | Y_k] \end{cases} \tag{6}$$

通过将式(2)带入式(3)可得

$$\boldsymbol{y}_{k+1}=(1-\gamma_{k+1})[h_{k+1}(\boldsymbol{x}_{k+1})+\boldsymbol{v}_{k+1}]+\gamma_{k+1}[h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k] \tag{7}$$

结合式(7)和式(6)中的 $\hat{\boldsymbol{y}}_{k+1|k}$ 及 $\boldsymbol{P}^{yy}_{k+1|k}$ 的相关定义,可以看出,计算 $\hat{\boldsymbol{y}}_{k+1|k}$ 及 $\boldsymbol{P}^{yy}_{k+1|k}$ 不仅仅依赖于状态估计量,还依赖于量测噪声的滤波估计量. 为了计算量测噪声 $\boldsymbol{v}_{k+1}$ 在 $k+1$ 时刻的滤波估计量,需要给出如下假设:

假设 2 假设量测噪声 $\boldsymbol{v}_{k+1}$ 和延迟量测 $\boldsymbol{y}_{k+1}$ 在延迟量测集合 Y_k 的条件下的联合分布服从 Student's t 分布,即

$$p(\boldsymbol{v}_{k+1},\boldsymbol{y}_{k+1}|Y_k)=\mathrm{St}\left(\begin{bmatrix}\boldsymbol{v}_{k+1}\\ \boldsymbol{y}_{k+1}\end{bmatrix};\begin{bmatrix}\boldsymbol{0}\\ \hat{\boldsymbol{y}}_{k+1|k}\end{bmatrix},\begin{bmatrix}\boldsymbol{R}_{k+1} & \boldsymbol{P}^{vy}_{k+1|k}\\ (\boldsymbol{P}^{vy}_{k+1|k})^{\mathrm{T}} & \boldsymbol{P}^{yy}_{k+1|k}\end{bmatrix},v_3\right) \tag{8}$$

注 1 依据 Student's t 分布的性质可知,式(8)若要成立,必须满足 $v_2=v_3$.

根据以上描述可知,由于量测一步随机延迟现象的存在,在推导带厚尾过程噪声和量测噪声的鲁棒 Student's t 滤波器框架时,需要同时考虑状态变量和噪声变量的更新,因此为便于处理,需要对状态进行扩维,构造一个新的状态向量:

$$\boldsymbol{x}^a_{k+1}=[\boldsymbol{x}^{\mathrm{T}}_{k+1}\boldsymbol{v}^{\mathrm{T}}_{k+1}] \tag{9}$$

则后验滤波概率密度函数 $p(\boldsymbol{x}^a_{k+1}|Y_{k+1})$ 可表示成如下形式:

$$p(\boldsymbol{v}_{k+1},\boldsymbol{x}_{k+1}|Y_{k+1})=\mathrm{St}\left(\begin{bmatrix}\boldsymbol{x}_{k+1}\\ \boldsymbol{v}_{k+1}\end{bmatrix};\begin{bmatrix}\hat{\boldsymbol{x}}_{k+1|k+1}\\ \hat{\boldsymbol{v}}_{k+1|k+1}\end{bmatrix},\begin{bmatrix}\boldsymbol{P}_{k+1|k+1} & \boldsymbol{P}^{xv}_{k+1|k+1}\\ (\boldsymbol{P}^{xv}_{k+1|k+1})^{\mathrm{T}} & \boldsymbol{P}^{yy}_{k+1|k}\end{bmatrix},v_3\right) \tag{10}$$

$$p(\boldsymbol{v}_{k+1},\boldsymbol{x}_{k+1}|Y_{k+1})=\mathrm{St}(\boldsymbol{x}^a;\hat{\boldsymbol{x}}^a_{k+1},\boldsymbol{P}_{k+1|k},v_3) \tag{11}$$

$$\hat{\boldsymbol{x}}^a_{k+1}=\begin{bmatrix}\hat{\boldsymbol{x}}_{k+1|k+1}\\ \hat{\boldsymbol{v}}_{k+1|k+1}\end{bmatrix}\boldsymbol{P}^a_{k+1|k+1}=\begin{bmatrix}\boldsymbol{P}_{k+1|k+1} & \boldsymbol{P}^{xv}_{k+1|k+1}\\ (\boldsymbol{P}^{xv}_{k+1|k+1})^{\mathrm{T}} & \boldsymbol{P}^{vv}_{k+1|k+1}\end{bmatrix} \tag{12}$$

其中 $\boldsymbol{P}^{xv}_{k+1|k+1}=\dfrac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1|k+1}\tilde{\boldsymbol{v}}^{\mathrm{T}}_{k+1|k+1}|Y_{k+1}]$.

式(10)~(12)可通过以下两个定理进行证明.

定理 1 给定 k 时刻的扩维状态量 $\boldsymbol{x}^a_k$ 和尺度矩阵 $\boldsymbol{P}^a_{k|k}$,在假设 2 的前提下,基于最小均方误差估计准则可知,后验 PDF $p(\boldsymbol{v}_{k+1}|Y_{k+1})$ 的滤波估计量 $\hat{\boldsymbol{v}}_{k+1|k+1}$ 及尺度矩阵 $\boldsymbol{P}^{vv}_{k+1|k+1}$ 可表示成如下形式:

$$\hat{\boldsymbol{v}}_{k+1|k+1}=\boldsymbol{K}^v_k(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k}) \tag{13}$$

$$\boldsymbol{P}^{vv}_{k+1|k+1}=\frac{(v_3-2)(v_3+\Delta^2_k)}{v_3(v_3'-2)}[\boldsymbol{R}_{k+1}-\boldsymbol{K}^v_k\boldsymbol{P}^{yy}_{k+1|k}(\boldsymbol{K}^v_k)^{\mathrm{T}}] \tag{14}$$

$$\boldsymbol{K}^v_k=\boldsymbol{P}^{vy}_{k+1|k}(\boldsymbol{P}^{yy}_{k+1|k})^{-1} \tag{15}$$

$$v_3'=v_3+m \tag{16}$$

$$\Delta^2_k=(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k})(\boldsymbol{P}^{yy}_{k+1|k})^{-1}(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k})^{\mathrm{T}} \tag{17}$$

$$\hat{\boldsymbol{y}}_{k+1|k}=(1-\lambda)\hat{z}_{k+1|k}+\lambda\hat{z}_{k|k} \tag{18}$$

$$\boldsymbol{P}^{yy}_{k+1|k+1}=(1-\lambda)\boldsymbol{P}^{zz}_{k+1|k}+\lambda\boldsymbol{P}^{zz}_{k|k}+\frac{v_3-2}{v_3}(1-\lambda)\lambda(z_{k+1}-\hat{z}_{k+1|k})(z_{k+1}-\hat{z}_{k+1|k})^{\mathrm{T}} \tag{19}$$

$$\boldsymbol{P}^{vy}_{k+1|k+1}=(1-\lambda)\boldsymbol{R}_{k+1} \tag{20}$$

其中,$\boldsymbol{K}^v_k$ 为滤波增益矩阵,并且

$$\hat{z}_{k+1|k}=\int h_{k+1}(\boldsymbol{x}_{k+1})\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1} \tag{21}$$

$$\boldsymbol{P}^{zz}_{k+1|k}=\frac{v_3-2}{v_3}\int h_{k+1}(\boldsymbol{x}_{k+1})h^{\mathrm{T}}_{k+1}(\boldsymbol{x}_{k+1})\times\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},\boldsymbol{v}_3)\mathrm{d}\boldsymbol{x}_{k+1}-\frac{v_3-2}{v_3}\hat{z}_{k+1|k}\hat{z}^{\mathrm{T}}_{k+1|k}+\boldsymbol{R}_{k+1} \tag{22}$$

$$\hat{z}_{k|k}=\int[h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k]\mathrm{St}(\boldsymbol{x}^a_k;\hat{\boldsymbol{x}}^a_{k|k},\boldsymbol{P}^a_{k|k},v_3)\mathrm{d}\boldsymbol{x}^a_k \tag{23}$$

$$\boldsymbol{P}^{zz}_{k|k}=\frac{v_3-2}{v_3}\int[h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k][h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k]^{\mathrm{T}}\times\mathrm{St}(\boldsymbol{x}^a_k;\hat{\boldsymbol{x}}^a_{k|k},\boldsymbol{P}^a_{k|k},v_3)\mathrm{d}\boldsymbol{x}^a_k-\frac{v_3-2}{v_3}\hat{z}_{k|k}\hat{z}^{\mathrm{T}}_{k|k} \tag{24}$$

证明 根据假设 2 可知,量测噪声 $\boldsymbol{v}_{k+1}$ 和延迟量测 $\boldsymbol{y}_{k+1}$ 在延迟量测集合 Y_k 的条件下的联合分布服从 Student's t 分布,即

$$p(\boldsymbol{v}_{k+1},\boldsymbol{y}_{k+1}|Y_k)=\mathrm{St}\left(\begin{bmatrix}\boldsymbol{v}_{k+1}\\ \boldsymbol{y}_{k+1}\end{bmatrix};\begin{bmatrix}\boldsymbol{0}\\ \hat{\boldsymbol{y}}_{k+1|k}\end{bmatrix},\begin{bmatrix}\boldsymbol{R}_{k+1} & \boldsymbol{P}^{vy}_{k+1|k}\\ (\boldsymbol{P}^{vy}_{k+1|k})^{\mathrm{T}} & \boldsymbol{P}^{yy}_{k+1|k}\end{bmatrix},v_3\right) \tag{25}$$

根据贝叶斯准则可得

$$p(\boldsymbol{v}_{k+1}|Y_{k+1})=\frac{p(\boldsymbol{v}_{k+1},\boldsymbol{y}_{k+1}|Y_k)}{p(\boldsymbol{y}_{k+1}|Y_k)}=\mathrm{St}(\boldsymbol{v}_{k+1};\hat{\boldsymbol{v}}'_{k+1|k+1},\boldsymbol{P}'^{vv}_{k+1|k+1},v_3') \tag{26}$$

式中 $\hat{\boldsymbol{v}}'_{k+1|k+1}$、$\boldsymbol{P}'^{vv}_{k+1|k+1}$、$v_3'$ 分别是后验概率密度函数 $p(\boldsymbol{v}_{k+1}|Y_{k+1})$ 的均值、尺度矩阵以及自由度参数.

根据文献[19] Student's t 分布的性质可得

$$\hat{\boldsymbol{v}}'_{k+1|k+1}=\boldsymbol{K}^v_k(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k}) \tag{27}$$

$$\boldsymbol{P}'^{vv}_{k+1|k+1}=\frac{v_3+\Delta^2_k}{v_3'}[\boldsymbol{R}_{k+1}-\boldsymbol{K}^v_k\boldsymbol{P}^{yy}_{k+1|k}(\boldsymbol{K}^v_k)^{\mathrm{T}}] \tag{28}$$

式中 Δ^2_k、v_3' 以及 $\boldsymbol{K}^v_k$ 分别如式(15)~(17)所示.

从式(16)可以看出随着时间的增加 $\boldsymbol{v}_3'$ 将趋向于正无穷,根据文献[19]可知,自由度参数 $\boldsymbol{v}_3'$ 趋向正无穷时,将不再服从 Student's t 分布,而是高斯分布,这必将导致 Student's t 滤波器的发散,为了保证 Student's t 滤波器的收敛性,可采用矩匹配方法[18]解决这个问题,如下所示:

$$\hat{\boldsymbol{v}}_{k+1|k+1}=\hat{\boldsymbol{v}}'_{k+1|k+1},\frac{v_3}{v_3-2}\boldsymbol{P}^{vv}_{k+1|k+1}=\frac{v_3'}{v_3'-2}\boldsymbol{P}'^{vv}_{k+1|k+1} \tag{29}$$

根据式(29),$\boldsymbol{P}^{vv}_{k+1|k+1}$可表示成如下形式:

$$\begin{aligned}\boldsymbol{P}^{vv}_{k+1|k+1}&=\frac{(v_3-2)v_3'}{v_3(v_3'-2)}\boldsymbol{P}'^{vv}_{k+1|k+1}\\&=\frac{(v_3-2)(v_3+\Delta_k^2)}{v_3(v_3+m-2)}[\boldsymbol{R}_{k+1}-\boldsymbol{K}^v_k\boldsymbol{P}^{yy}_{k+1|k}(\boldsymbol{K}^v_k)^{\mathrm{T}}]\end{aligned} \tag{30}$$

则 $p(\boldsymbol{v}_{k+1}|Y_{k+1})$就相应的转换成如下分布:

$$p(\boldsymbol{v}_{k+1}|Y_{k+1})=\mathrm{St}(\boldsymbol{v}_{k+1};\hat{\boldsymbol{v}}_{k+1|k+1},\boldsymbol{P}^{vv}_{k+1|k+1},v_3) \tag{31}$$

根据全概率准则可知

$$\begin{aligned}p(\boldsymbol{x}_{k+1}|Y_k)&=\int p(\boldsymbol{x}_{k+1}|\boldsymbol{x}_k)p(\boldsymbol{x}_k|Y_k)\mathrm{d}\boldsymbol{x}_k\\&=\int\mathrm{St}(\boldsymbol{x}_{k+1};f_k(\boldsymbol{x}_k),\boldsymbol{Q}_k,v_1)\\&\quad\times\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k\end{aligned} \tag{32}$$

从式(32)可知 $p(\boldsymbol{x}_{k+1}|Y_k)$由于非线性 $f_k(\cdot)$传递导致其不再满足 Student's t 分布,因此依据假设 1 需要对 $p(\boldsymbol{x}_{k+1}|Y_k)$进行如下近似:

$$p(\boldsymbol{x}_{k+1}|Y_k)=\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3) \tag{33}$$

根据全概率准则和注 1 可得:

$$\begin{aligned}p(\boldsymbol{z}_{k+1}|Y_k)&=\int p(\boldsymbol{z}_{k+1}|\boldsymbol{x}_{k+1})p(\boldsymbol{x}_{k+1}|Y_k)\mathrm{d}\boldsymbol{x}_{k+1}\\&=\int\mathrm{St}(\boldsymbol{z}_{k+1};h_{k+1}(\boldsymbol{x}_{k+1}),\boldsymbol{R}_{k+1},v_2)\\&\quad\times\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1}\\&=\mathrm{St}(\boldsymbol{z}_{k+1};\hat{\boldsymbol{z}}_{k+1|k},\boldsymbol{P}^{zz}_{k+1|k},v_3)\end{aligned} \tag{34}$$

则在最小均方误差估计意义下,根据 Student's t 分布的性质,结合量测式(2),可求得$\hat{\boldsymbol{z}}_{k+1|k}$和 $\boldsymbol{P}^{zz}_{k+1|k}$:

$$\begin{aligned}\hat{\boldsymbol{z}}_{k+1|k}&=E[\boldsymbol{z}_{k+1|k}|Y_k]=E[(h_{k+1}(\boldsymbol{x}_{k+1})+\boldsymbol{v}_{k+1})|Y_k]\\&=\int(h_{k+1}(\boldsymbol{x}_{k+1})+\boldsymbol{v}_{k+1})\\&\quad\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1}\\&=\int h_{k+1}(\boldsymbol{x}_{k+1})\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1}\end{aligned} \tag{35}$$

$$\begin{aligned}\boldsymbol{P}^{zz}_{k+1|k}&=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{z}}_{k+1|k}\tilde{\boldsymbol{z}}^{\mathrm{T}}_{k+1|k}|Y_k]\\&=\frac{v_3-2}{v_3}\{E[\boldsymbol{z}_{k+1}\boldsymbol{z}^{\mathrm{T}}_{k+1}|Y_k]-\hat{\boldsymbol{z}}_{k+1|k}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k+1|k}\}\\&=\frac{v_3-2}{v_3}\int h_{k+1}(\boldsymbol{x}_{k+1})h^{\mathrm{T}}_{k+1}(\boldsymbol{x}_{k+1})\\&\quad\times\mathrm{St}(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1}\\&\quad-\frac{v_3-2}{v_3}\hat{\boldsymbol{z}}_{k+1|k}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k+1|k}+\boldsymbol{R}_{k+1}\end{aligned} \tag{36}$$

类似的可以求得 $\hat{\boldsymbol{z}}_{k|k}$和 $\boldsymbol{P}^{zz}_{k|k}$,如下:

$$\hat{\boldsymbol{z}}_{k|k}=\int h_k(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k \tag{37}$$

$$\begin{aligned}\boldsymbol{P}^{zz}_{k|k}&=\frac{v_3-2}{v_3}\int[h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k][h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k]^{\mathrm{T}}\\&\quad\times\mathrm{St}(\boldsymbol{x}^a_k;\hat{\boldsymbol{x}}^a_{k|k},\boldsymbol{P}^a_{k|k},v_3)\mathrm{d}\boldsymbol{x}^a_k-\frac{v_3-2}{v_3}\hat{\boldsymbol{z}}_{k|k}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k|k}\end{aligned} \tag{38}$$

根据式(2)~(4)可知,量测一步预测估计误差可表示成如下形式:

$$\begin{aligned}\tilde{\boldsymbol{y}}_{k+1|k}&=\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k}\\&=(1-\gamma_{k+1})\tilde{\boldsymbol{z}}_{k+1|k}+\gamma_{k+1}\tilde{\boldsymbol{z}}_{k|k}\\&\quad+(\gamma_{k+1}-\lambda)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k+1|k})\end{aligned} \tag{39}$$

通过将式(39)带入式(6)中关于 $\hat{\boldsymbol{y}}_{k+1|k}$和 $\boldsymbol{P}^{yy}_{k+1|k}$的相关定义中,可求得

$$\begin{aligned}\hat{\boldsymbol{y}}_{k+1|k}&=E[\boldsymbol{y}_{k+1}|Y_k]=E[(1-\gamma_{k+1})\boldsymbol{z}_{k+1}+\gamma_{k+1}\boldsymbol{z}_k|Y_k]\\&=(1-\lambda)E[\boldsymbol{z}_{k+1}|Y_k]+\lambda E[\boldsymbol{z}_k|Y_k]\\&=(1-\lambda)\hat{\boldsymbol{z}}_{k+1|k}+\lambda\hat{\boldsymbol{z}}_{k|k}\end{aligned} \tag{40}$$

$$\begin{aligned}\boldsymbol{P}^{yy}_{k+1|k}&=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{y}}_{k+1|k}\tilde{\boldsymbol{y}}^{\mathrm{T}}_{k+1|k}|Y_k]\\&=\frac{v_3-2}{v_3}\{E[(1-\gamma_{k+1})^2\tilde{\boldsymbol{z}}_{k+1|k}\tilde{\boldsymbol{z}}^{\mathrm{T}}_{k+1|k}|Y_k]\\&\quad+E[\gamma^2_{k+1}\tilde{\boldsymbol{z}}_{kk}\tilde{\boldsymbol{z}}^{\mathrm{T}}_{k|k}|Y_k]\\&\quad+E[(\gamma_{k+1}-\lambda)(\gamma_{k+1}-\lambda)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k+1|k})\\&\quad(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k+1|k})^{\mathrm{T}}|Y_k]\}\\&=(1-\lambda)\boldsymbol{P}^{zz}_{k+1|k}+\lambda\boldsymbol{P}^{zz}_{k|k}\\&\quad+\frac{v_3-2}{v_3}(1-\lambda)\lambda(\hat{\boldsymbol{z}}_k-\hat{\boldsymbol{z}}_{k+1|k})(\hat{\boldsymbol{z}}_k-\hat{\boldsymbol{z}}_{k+1|k})^{\mathrm{T}}\end{aligned} \tag{41}$$

同理可求得 $\boldsymbol{P}^{vy}_{k+1|k}$:

$$\boldsymbol{P}^{vy}_{k+1|k}=\frac{v_3-2}{v_3}E[\boldsymbol{v}_{k+1}\tilde{\boldsymbol{y}}^{\mathrm{T}}_{k+1|k}|Y_k]=(1-\lambda)\boldsymbol{R}_{k+1} \tag{42}$$

定理 2　给定 k 时刻的扩维状态量 $\boldsymbol{x}^a_k$ 和尺度矩阵 $\boldsymbol{P}^a_{k|k}$,在假设 1 的前提下,基于最小均方误差估计准则,后验 PDF$p(\boldsymbol{x}_{k+1}|Y_{k+1})$的 Student's t 近似估计的滤波估计量 $\hat{\boldsymbol{x}}_{k+1|k+1}$及尺度矩阵 $\boldsymbol{P}_{k+1|k+1}$可表示成如下形式:

$$\hat{\boldsymbol{x}}_{k+1|k+1}=\hat{\boldsymbol{x}}_{k+1|k}+\boldsymbol{K}^x_k(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k}) \tag{43}$$

$$\boldsymbol{P}_{k+1|k+1}=\frac{(v_3-2)(v_3+\Delta_k^2)}{v_3(v_3'-2)}[\boldsymbol{P}_{k+1|k}-\boldsymbol{K}^x_k\boldsymbol{P}^{yy}_{k+1|k}(\boldsymbol{K}^x_k)^{\mathrm{T}}] \tag{44}$$

$$\boldsymbol{K}^x_k=\boldsymbol{P}^{xy}_{k+1|k}(\boldsymbol{P}^{yy}_{k+1|k})^{-1} \tag{45}$$

$$v_3'=v_3+m \tag{46}$$

$$\Delta_k^2=(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k})(\boldsymbol{P}^{yy}_{k+1|k})^{-1}(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k})^{\mathrm{T}} \tag{47}$$

$$\boldsymbol{P}^{xy}_{k+1|k}=(1-\lambda)\boldsymbol{P}^{xz}_{k+1|k}+\lambda\boldsymbol{P}^{xz}_{k+1,k|k} \tag{48}$$

$$\boldsymbol{P}^{xv}_{k+1|k+1}=-\boldsymbol{K}^x_k\boldsymbol{P}^{xy}_{k+1|k}(\boldsymbol{K}^v_k)^{\mathrm{T}} \tag{49}$$

其中 $\boldsymbol{K}^x_k$ 为滤波增益矩阵,并且

$$\hat{\boldsymbol{x}}_{k+1|k}=\int f_k(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k \tag{50}$$

$$\boldsymbol{P}_{k+1|k}=\frac{v_3-2}{v_3}\int f_k(\boldsymbol{x}_k)f^{\mathrm{T}}_k(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k$$

$$-\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{x}}_{k+1|k}^{\mathrm{T}}+\frac{v_1(v_3-2)}{(v_1-2)v_3}\boldsymbol{Q}_k \quad (51)$$

$$\boldsymbol{P}_{k+1,k|k}^{xz}=\frac{v_3-2}{v_3}\int f_k(\boldsymbol{x}_k)[h_k(\boldsymbol{x}_k)+\boldsymbol{v}_k]^{\mathrm{T}} \times \mathrm{St}(\boldsymbol{x}_k;\hat{x}_{k|k}^a,\boldsymbol{P}_{k|k}^a,v_3)\mathrm{d}\boldsymbol{x}_{k|k}^a-\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k|k}^{\mathrm{T}} \quad (52)$$

$$\boldsymbol{P}_{k+1|k}^{xz}=\frac{v_3-2}{v_3}\int \boldsymbol{x}_{k+1}h_k^{\mathrm{T}}(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1} -\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k+1|k}^{\mathrm{T}} \quad (53)$$

证明 根据假设1可知，$k+1$ 时刻状态量 $\boldsymbol{x}_{k+1}$ 和延迟量测 $\boldsymbol{y}_{k+1}$ 在延迟量测集合 Y_k 的条件下的联合分布服从 Student's t 分布，即

$$p(\boldsymbol{x}_{k+1},\boldsymbol{y}_{k+1}|Y_k)=\mathrm{St}\left(\begin{bmatrix}\boldsymbol{x}_{k+1}\\ \boldsymbol{y}_{k+1}\end{bmatrix};\begin{bmatrix}\hat{\boldsymbol{x}}_{k+1|k}\\ \hat{\boldsymbol{y}}_{k+1|k}\end{bmatrix},\begin{bmatrix}\boldsymbol{P}_{k+1|k} & \boldsymbol{P}_{k+1|k}^{xy}\\ (\boldsymbol{P}_{k+1|k}^{xy})^{\mathrm{T}} & \boldsymbol{P}_{k+1|k}^{yy}\end{bmatrix},v_3\right) \quad (54)$$

根据贝叶斯准则可得

$$p(\boldsymbol{x}_{k+1}|Y_{k+1})=\frac{p(\boldsymbol{x}_{k+1},\boldsymbol{y}_{k+1}|Y_k)}{p(\boldsymbol{y}_{k+1}|Y_k)} =\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}'_{k+1|k+1},\boldsymbol{P}'_{k+1|k+1},v'_3) \quad (55)$$

式中 $\hat{\boldsymbol{x}}'_{k+1|k+1}$、$\boldsymbol{P}'_{k+1|k+1}$、$v'_3$ 分别后验概率密度函数 $p(\boldsymbol{x}_{k+1}|Y_{k+1})$ 的均值、尺度矩阵以及自由度参数.

根据文献[19]Student's t 分布的性质可知

$$\hat{\boldsymbol{x}}'_{k+1|k+1}=\hat{\boldsymbol{x}}_{k+1|k}+\boldsymbol{K}_k^x(\boldsymbol{y}_{k+1}-\hat{\boldsymbol{y}}_{k+1|k}) \quad (56)$$

$$\boldsymbol{P}'_{k+1|k+1}=\frac{v_3+\Delta_k^2}{v'_3}[\boldsymbol{P}_{k+1|k}-\boldsymbol{K}_k^x\boldsymbol{P}_{k+1|k}^{yy}(\boldsymbol{K}_k^x)^{\mathrm{T}}] \quad (57)$$

式中 Δ_k^2、v'_3 以及 $\boldsymbol{K}_k^x$ 分别如式(45)~(47)所示.

同定理1可知，$\boldsymbol{P}_{k+1|k+1}$ 可表示成如下形式：

$$\boldsymbol{P}_{k+1|k+1}=\frac{(v_3-2)(v_3+\Delta_k^2)}{v_3(v'_3-2)}[\boldsymbol{R}_{k+1}-\boldsymbol{K}_k^v\boldsymbol{P}_{k+1|k}^{yy}(\boldsymbol{K}_k^v)^{\mathrm{T}}] \quad (58)$$

则 $p(\boldsymbol{x}_{k+1}|Y_{k+1})$ 就相应的转换成如下分布：

$$p(\boldsymbol{x}_{k+1}|Y_{k+1})=\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k+1},\boldsymbol{P}_{k+1|k+1},v_3) \quad (59)$$

其中 $\hat{\boldsymbol{x}}_{k+1|k}$、$\boldsymbol{P}_{k+1|k}$、$\boldsymbol{P}_{k+1,k|k}^{xz}$ 可通过下式计算获得：

$$\hat{\boldsymbol{x}}_{k+1|k}=E[(f_k(\boldsymbol{x}_k)+\boldsymbol{w}_k)|Y_k] =\int f_k(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k \quad (60)$$

$$\boldsymbol{P}_{k+1|k}=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{x}}_{k+1|k}^{\mathrm{T}}|Y_k] =\frac{v_3-2}{v_3}\int f_k(\boldsymbol{x}_k)f_k^{\mathrm{T}}(\boldsymbol{x}_k)\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\mathrm{d}\boldsymbol{x}_k -\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{x}}_{k+1|k}^{\mathrm{T}}+\frac{v_1(v_3-2)}{(v_1-2)v_3}\boldsymbol{Q}_k \quad (61)$$

$$\boldsymbol{P}_{k+1,k|k}^{xz}=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1}\tilde{\boldsymbol{z}}_{k|k}^{\mathrm{T}}|Y_k] =\frac{v_3-2}{v_3}\{E[\boldsymbol{x}_{k+1}\boldsymbol{z}_{k|k}^{\mathrm{T}}|Y_k]-\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k|k}^{\mathrm{T}}\} =\frac{v_3-2}{v_3}\int f_k(\boldsymbol{x}_k)[h_k(\boldsymbol{x}_k)+v_k]^{\mathrm{T}} \times\mathrm{St}(\boldsymbol{x}_k;\hat{x}_{k|k}^a,\boldsymbol{P}_{k|k}^a,v_3)\mathrm{d}\boldsymbol{x}_{k|k}^a-\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k|k}^{\mathrm{T}} \quad (62)$$

$$\boldsymbol{P}_{k+1|k}^{xz}=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1}\tilde{\boldsymbol{z}}_{k+1|k}^{\mathrm{T}}|Y_k] =\frac{v_3-2}{v_3}\{E[\boldsymbol{x}_{k+1}\boldsymbol{z}_{k+1|k}^{\mathrm{T}}|Y_k]-\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k+1|k}^{\mathrm{T}}\} =\frac{v_3-2}{v_3}\int \boldsymbol{x}_{k+1}h_{k+1}^{\mathrm{T}}(\boldsymbol{x}_{k+1}) \times\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\mathrm{d}\boldsymbol{x}_{k+1} -\frac{v_3-2}{v_3}\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{z}}_{k+1|k}^{\mathrm{T}} \quad (63)$$

根据 $\boldsymbol{P}_{k+1|k}^{xy}$ 的相关定义可知

$$\boldsymbol{P}_{k+1|k}^{xy}=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{y}}_{k+1|k}^{\mathrm{T}}|Y_k] =\frac{v_3-2}{v_3}\{E[(1-\gamma_{k+1})\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{z}}_{k+1|k}|Y_k] +E[\gamma_{k+1}\tilde{\boldsymbol{x}}_{k+1|k}\tilde{\boldsymbol{z}}_{k|k}|Y_k] +E[(\gamma_{k+1}-\lambda)\tilde{\boldsymbol{x}}_{k+1|k}(z_k-\hat{\boldsymbol{z}}_{k+1|k})|Y_k]\} =(1-\lambda)\boldsymbol{P}_{k+1|k}^{xz}+\lambda\boldsymbol{P}_{k+1,k|k}^{xz} \quad (64)$$

根据式(13)和式(43)$\hat{\boldsymbol{x}}_{k+1|k+1}$ 和 $\hat{\boldsymbol{x}}_{k+1|k+1}$，可知

$$\tilde{\boldsymbol{v}}_{k+1|k+1}=\boldsymbol{v}_{k+1}-\boldsymbol{K}_k^v\tilde{\boldsymbol{y}}_{k+1|k} \quad (65)$$

$$\tilde{\boldsymbol{x}}_{k+1|k+1}=\boldsymbol{x}_{k+1}-\hat{\boldsymbol{x}}_{k+1|k}-\boldsymbol{K}_k^x\tilde{\boldsymbol{y}}_{k+1|k}=\tilde{\boldsymbol{x}}_{k+1|k}-\boldsymbol{K}_k^x\tilde{\boldsymbol{y}}_{k+1|k} \quad (66)$$

根据 $\boldsymbol{P}_{k+1|k+1}^{xv}$ 的相关定义可得

$$\boldsymbol{P}_{k+1|k+1}^{xv}=\frac{v_3-2}{v_3}E[\tilde{\boldsymbol{x}}_{k+1|k+1}\tilde{\boldsymbol{v}}_{k+1|k+1}^{\mathrm{T}}|Y_{k+1}] =\frac{v_3-2}{v_3}\{E[(\tilde{\boldsymbol{x}}_{k+1|k}-\boldsymbol{K}_k^x\tilde{\boldsymbol{y}}_{k+1|k})(\boldsymbol{v}_{k+1}-\tilde{\boldsymbol{y}}_{k+1|k})^{\mathrm{T}}|Y_{k+1}]\} =-\boldsymbol{P}_{k+1|k}^{xy}(\boldsymbol{K}_k^v)^{\mathrm{T}}-\boldsymbol{K}_k^x(\boldsymbol{P}_{k+1|k}^{vy})^{\mathrm{T}}+\boldsymbol{K}_k^x\boldsymbol{P}_{k+1|k}^{yy}(\boldsymbol{K}_k^v)^{\mathrm{T}} \quad (67)$$

又

$$\boldsymbol{K}_k^x\boldsymbol{P}_{k+1|k}^{xy}(\boldsymbol{K}_k^v)^{\mathrm{T}}=\boldsymbol{P}_{k+1|k}^{xy}(\boldsymbol{K}_k^v)^{\mathrm{T}}=\boldsymbol{K}_k^x(\boldsymbol{P}_{k+1|k}^{vy})^{\mathrm{T}} \quad (68)$$

则

$$\boldsymbol{P}_{k+1|k+1}^{xv}=-\boldsymbol{K}_k^x\boldsymbol{P}_{k+1|k}^{xy}(\boldsymbol{K}_k^v)^{\mathrm{T}} \quad (69)$$

得证.

由式(23)、式(50)可知，$p(\boldsymbol{v}_{k+1}|Y_{k+1})$ 和 $p(\boldsymbol{x}_{k+1}|Y_{k+1})$ 均服从 Student's t 分布，且自由度参数均为 $\boldsymbol{v}_3$，则依据 Student's t 分布的性质可求得式(10)~(12).

注2 当延迟概率 $\lambda=0$ 时，根据式(2)、(3)可知，$\boldsymbol{y}_k=z_k$，则定理2中的式(33)~(35)可转换成如下形式：

$$\begin{cases}\hat{\boldsymbol{x}}_{k+1|k+1}=\hat{\boldsymbol{x}}_{k+1|k}+\boldsymbol{K}_k^x(z_{k+1}-\hat{z}_{k+1|k})\\ \boldsymbol{P}_{k+1|k+1}=\dfrac{(v_3-2)(v_3+\Delta_k^2)}{v_3(v_3'-2)}[\boldsymbol{P}_{k+1|k}-\boldsymbol{K}_k^x\boldsymbol{P}_{k+1|k}^{zz}(\boldsymbol{K}_k^x)^{\mathrm{T}}]\\ \boldsymbol{K}_k^x=\boldsymbol{P}_{k+1|k}^{xz}(\boldsymbol{P}_{k+1|k}^{zz})^{-1}\end{cases}\tag{70}$$

当 $v_1=v_2=v_3=v\to+\infty$时,则

$$\begin{cases}\mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k},v_3)\to N(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k})\\ \mathrm{St}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},v_3)\to N(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k})\end{cases}\tag{71}$$

则 $\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},\hat{z}_{k+1|k},\boldsymbol{P}_{k+1|k}^{zz},\boldsymbol{P}_{k+1|k}^{xz}$可化成如下形式:

$$\lim_{v\to\infty}\hat{\boldsymbol{x}}_{k+1|k}=\int f_k(\boldsymbol{x}_k)N(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k})\mathrm{d}\boldsymbol{x}_k\tag{72}$$

$$\lim_{v\to\infty}\boldsymbol{P}_{k+1|k}=\int f_k(\boldsymbol{x}_k)f_k^{\mathrm{T}}(\boldsymbol{x}_k)N(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k},\boldsymbol{P}_{k|k})\mathrm{d}\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{x}}_{k+1|k}^{\mathrm{T}}+\boldsymbol{Q}_k\tag{73}$$

$$\lim_{v\to\infty}\hat{z}_{k+1|k}=\int h_{k+1}(\boldsymbol{x}_{k+1})N(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k})\mathrm{d}\boldsymbol{x}_{k+1}\tag{74}$$

$$\lim_{v\to\infty}\boldsymbol{P}_{k+1|k}^{zz}=\int h_{k+1}(\boldsymbol{x}_{k+1})h_{k+1}^{\mathrm{T}}(\boldsymbol{x}_{k+1})N(\boldsymbol{x}_{k+1};\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k})\mathrm{d}\boldsymbol{x}_{k+1}-\hat{z}_{k+1|k}\hat{z}_{k+1|k}^{\mathrm{T}}+\boldsymbol{R}_{k+1}\tag{75}$$

$$\lim_{v\to\infty}\boldsymbol{P}_{k+1|k}^{xz}=\int \boldsymbol{x}_{k+1}h_{k+1}^{\mathrm{T}}(\boldsymbol{x}_{k+1})N(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k})\mathrm{d}\boldsymbol{x}_{k+1}-\hat{\boldsymbol{x}}_{k+1|k}\hat{z}_{k+1|k}^{\mathrm{T}}\tag{76}$$

结合式(71)~(77)可知,当 $v_1=v_2=v_3=v\to+\infty$且 $\lambda=0$ 时,本文提出的算法就变成了高斯近似滤波算法,也就是说高斯近似滤波器只是本文的一个特殊情况.

4　量测随机延迟下带厚尾过程噪声和量测噪声的鲁棒 Student's t 随机容积滤波器

本文采用了文献[25,28]中所提出的随机 Student's t-球面相径容积规则进行非线性积分计算,在此基础上提出了基于随机 Student's t-球面相径容积规则的量测随机延迟下带厚尾过程噪声和量测噪声的鲁棒 Student's t 随机容积滤波器,具体的实现过程如下所示:,

算法1　量测随机延迟下带厚尾噪声的鲁棒 Student's t 随机容积滤波器

1:给定 $\hat{\boldsymbol{x}}_0^a$、$\boldsymbol{P}_0^a$、$\hat{x}_{k|k}^a$,$\boldsymbol{P}_{k|k}^a$、$\boldsymbol{Q}_k$、$\boldsymbol{R}_{k+1}$、v_1、v_2、v_3 $f_k(\cdot)$、

2:$h_k(\cdot)$、λ、n、N

时间更新:

3: $\boldsymbol{\xi}_i=[\boldsymbol{0},\{\mathbf{e}_i\}_{i=2}^{n+m+1},\{-\mathbf{e}_{i-n-m-1}\}_{i=n+m+2}^{2(n+m)+1}]$

4:For $j=1:N$

5:　产生 $n\times n$ 的服从正态分布的矩阵$\boldsymbol{U}_j$,用过 QR

6:　分解产生正交矩阵 $\boldsymbol{Q}_j$,$\boldsymbol{U}_j=\boldsymbol{Q}_j\boldsymbol{R}_j$.

7:　产生随机变量 $\tau\sim\mathrm{Beta}(\tau;\frac{n+2}{2},\frac{v-2}{2})$

8:　计算容积点 $\boldsymbol{\xi}_{i,k|k}^a(i=1,2,\cdots,2(n+m),2(n+m)+1)$

9:　$\boldsymbol{\xi}_{i,k|k}^a=[(\boldsymbol{\xi}_{i,k|k}^x)^{\mathrm{T}}(\boldsymbol{\xi}_{i,k|k}^v)^{\mathrm{T}}]^{\mathrm{T}}=r_{2,j}\sqrt{v_3\boldsymbol{P}_{k|k}^a}\boldsymbol{Q}_j\boldsymbol{\xi}_i+\hat{\boldsymbol{x}}_{k|k}^a$

10:　计算容积点的权重 w_i

11: $w_i=\begin{cases}1-\dfrac{n}{(v_3-2)r_{2,j}^2},i=1\\ \dfrac{n}{2(v_3-2)r_{2,j}^2},i=2,3,\cdots,2(n+m)+1\end{cases}$

12:　计算经状态方程传递的容积点

13:　$\boldsymbol{\chi}_{i,k+1|k}^x=f_k(\boldsymbol{\xi}_{i,k|k}^x),(i=1,2,\cdots,2n,2(n+m)+1)$

14:　计算 $\hat{\boldsymbol{x}}_{k+1|k}^j,\boldsymbol{P}_{k+1|k}^j$

15: $\hat{\boldsymbol{x}}_{k+1|k}^j=\sum_{i=1}^{2(n+m)+1}w_i\boldsymbol{\chi}_{i,k+1|k}^x,\boldsymbol{P}_{k+1|k}^j=\sum_{i=1}^{2(n+m)+1}w_i\boldsymbol{\xi}_{i,k|k}^x(\boldsymbol{\xi}_{i,k|k}^x)^{\mathrm{T}}$

16:END FOR

17: $\hat{\boldsymbol{x}}_{k+1|k}=\frac{1}{N}\sum_{j=1}^{N}\hat{\boldsymbol{x}}_{k+1|k}^j,\boldsymbol{P}_{k+1|k}=\frac{v_3-2}{v_3}\left[\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{P}_{k+1|k}^j-\hat{\boldsymbol{x}}_{k+1|k}\hat{\boldsymbol{x}}_{k+1|k}^{\mathrm{T}}\right]+\frac{v_1(v_3-2)}{(v_1-2)v_3}\boldsymbol{Q}_k$

量测更新:

18:For $j=1:N$

19:　重复 5~7,生成 $\boldsymbol{Q}_j$ 和随机变量 τ

20:　计算容积点 $\boldsymbol{\xi}_{i,k+1|k}(i=1,2,\cdots,2n,2n+1)$

21:　$\boldsymbol{\xi}_{i,k+1|k}=r_{2,j}\sqrt{v_3\boldsymbol{P}_{k+1|k}}\boldsymbol{Q}_j\boldsymbol{\xi}_i+\hat{\boldsymbol{x}}_{k+1|k}$

22:　容积点 $\boldsymbol{\xi}_{i,k+1|k}$的权重为 w_i'

23　$w_i'=\begin{cases}1-\dfrac{n}{(v_3-2)r_{2,j}^2},i=1\\ \dfrac{n}{2(v_3-2)r_{2,j}^2},i=2,3,\cdots,2n+1\end{cases}$

24:　计算经量测方程传递的容积点

25:　$\boldsymbol{\eta}_{i,k+1|k}^v=h_k(\boldsymbol{\xi}_{i,k|k}^v),\boldsymbol{\theta}_{i,k+1|k}=h_{k+1}(\boldsymbol{\xi}_{i,k+1|k})$

26:　计算 $\hat{z}_{k|k}^j,\boldsymbol{P}_{k|k}^{j,zz},\hat{z}_{k+1|k}^j,\boldsymbol{P}_{k+1|k}^{j,zz},\boldsymbol{P}_{k+1|k}^{j,xz},\boldsymbol{P}_{k+1,k|k}^{j,xz}$

27:　$\hat{z}_{k|k}^j=\sum_{i=1}^{2(n+m)+1}w_i\boldsymbol{\eta}_{i,k+1|k}^v\boldsymbol{P}_{k|k}^{j,zz}=\sum_{i=1}^{2(n+m)+1}w_i(\boldsymbol{\eta}_{i,k+1|k}^v+\boldsymbol{\xi}_{i,k|k}^v)(\boldsymbol{\eta}_{i,k+1|k}^v+\boldsymbol{\xi}_{i,k|k}^v)^{\mathrm{T}}$

28:　$\hat{z}_{k+1|k}^j=\sum_{i=1}^{2n+1}w_i'\boldsymbol{\theta}_{i,k+1|k}\boldsymbol{P}_{k+1|k}^{j,zz}=\sum_{i=1}^{2n+1}w_i'\boldsymbol{\theta}_{i,k+1|k}(\boldsymbol{\theta}_{i,k+1|k})^{\mathrm{T}}$

29:　$\boldsymbol{P}_{k+1|k}^{j,xz}=\sum_{i=1}^{2n+1}w_i'\boldsymbol{\xi}_{i,k+1|k}(\boldsymbol{\theta}_{i,k+1|k})^{\mathrm{T}}$

30:　$\boldsymbol{P}_{k+1,k|k}^{j,xz}=\sum_{i=1}^{2(n+m)+1}w_i\boldsymbol{\chi}_{i,k+1|k}^x(\boldsymbol{\eta}_{i,k+1|k}^v+\boldsymbol{\xi}_{i,k|k}^v)^{\mathrm{T}}$

31:END FOR

32: $\hat{z}_{k|k}=\frac{1}{N}\sum_{j=1}^{N}\hat{z}_{k|k}^j,\hat{z}_{k+1|k}=\frac{1}{N}\sum_{j=1}^{N}\hat{z}_{k+1|k}^j$

33: $\boldsymbol{P}_{k|k}^{zz}=\frac{v_3-2}{v_3}\left[\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{P}_{k|k}^{j,zz}-\hat{\boldsymbol{x}}_{k|k}\hat{z}_{k|k}^{\mathrm{T}}\right]$

34: $\boldsymbol{P}_{k+1|k}^{zz}=\frac{v_3-2}{v_3}\left[\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{P}_{k+1|k}^{j,zz}-\hat{z}_{k+1|k}\hat{z}_{k+1|k}^{\mathrm{T}}\right]+\boldsymbol{R}_{k+1}$

35: $\boldsymbol{P}_{k+1|k}^{xz}=\frac{v_3-2}{v_3}\left[\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{P}_{k|k}^{j,xz}-\hat{\boldsymbol{x}}_{k+1|k}\hat{z}_{k+1|k}^{\mathrm{T}}\right]$

36: $\boldsymbol{P}_{k+1,k|k}^{xz}=\frac{v_3-2}{v_3}\left[\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{P}_{k+1,k|k}^{j,xz}-\hat{\boldsymbol{x}}_{k+1|k}\hat{z}_{k|k}^{\mathrm{T}}\right]$

37:将 $\hat{z}_{k|k},\boldsymbol{P}_{k|k}^{zz},\hat{z}_{k+1|k},\boldsymbol{P}_{k+1|k}^{zz}$,带入定理 1 中求得 $\hat{\boldsymbol{x}}_{k+1|k+1},\boldsymbol{P}_{k+1|k+1}^{vv}$

38:将 $\hat{\boldsymbol{x}}_{k+1|k},\boldsymbol{P}_{k+1|k},\hat{z}_{k|k},\boldsymbol{P}_{k|k}^{zz},\hat{z}_{k+1|k},\boldsymbol{P}_{k+1|k}^{zz},\boldsymbol{P}_{k+1|k}^{xz},\boldsymbol{P}_{k+1,k|k}^{xz}$ 到定理 2 中求得

39: $\hat{\boldsymbol{x}}_{k+1|k+1},\boldsymbol{P}_{k+1|k+1},\boldsymbol{P}_{k+1|k+1}^{xv}$.

5　仿真分析

本文利用带有未知角速度突变的协同转弯机动目标跟踪仿真验证本文算法的有效性和优越性,其系统方程为:

$$\boldsymbol{x}_k=\begin{bmatrix}1 & \dfrac{\sin(\Omega T)}{\Omega} & 0 & \dfrac{\cos(\Omega T)-1}{\Omega}\\ 0 & \cos(\Omega T) & 0 & -\sin(\Omega T)\\ 0 & \dfrac{1-\cos(\Omega T)}{\Omega} & 1 & \dfrac{\sin(\Omega T)}{\Omega}\\ 0 & \sin(\Omega T) & 0 & \cos(\Omega T)\end{bmatrix}\cdot\boldsymbol{x}_{k-1}+\boldsymbol{w}_{k-1}$$

式中，$\boldsymbol{x}_k=[x_k,\dot{x}_k,y_k,\dot{y}_k,\Omega_k]^{\mathrm{T}}$ 为 k 时刻的目标状态；x_k 和 y_k 分别表示 x 和 y 方向的位置，$\dot{x}$ 和 $\dot{y}$ 分别表示 x 和 y 方向的速度；Ω_k 为未知的转弯角速度；采样周期 $T=1\mathrm{s}$；过程噪声 $\boldsymbol{w}_k\in\mathbb{R}^n$ 可通过两个高斯分布混合产生，如下所示：

$$p(\boldsymbol{w}_k)=(1-p)N(\boldsymbol{w}_k;\boldsymbol{0},\boldsymbol{Q})+pN(\boldsymbol{w}_k;\boldsymbol{0},50\boldsymbol{Q})$$

其中，$\boldsymbol{Q}=\mathrm{blkdiag}(q_1\boldsymbol{M},q_1\boldsymbol{M},q_2T)$，

$$\boldsymbol{M}=\begin{bmatrix}T^3/3 & T^2/2\\ T^2/2 & T\end{bmatrix}$$

$q_1=0.1\mathrm{m}^2/\mathrm{s}^3$，$q_2=1.75\times10^{-4}/\mathrm{s}^3$，$p$ 为污染度，在此定为 0.05.

通过载机雷达可以获得目标与载机之间的相对距离 r、方向角 φ 的信息，则可获得系统的非线性量测方程为：

$$\boldsymbol{z}_k=h(\boldsymbol{x}_k)+\boldsymbol{v}_k=\begin{bmatrix}\sqrt{x_k^2+y_k^2}\\ \arctan(y_k/x_k)\end{bmatrix}+\boldsymbol{v}_k$$

式中：量测噪声 $\boldsymbol{v}_k\in\mathbb{R}^m$ 可通过两个高斯分布混合产生，如下所示：

$$p(\boldsymbol{v}_k)=(1-p)N(\boldsymbol{v}_k;\boldsymbol{0},\boldsymbol{R})+pN(\boldsymbol{v}_k;\boldsymbol{0},50\boldsymbol{R})$$

其中，$\boldsymbol{R}=[\sigma_r^2\quad\sigma_\varphi^2]$，$\sigma_r=10\mathrm{m}$，$\sigma_\varphi=\sqrt{10}\times10^{-3}\mathrm{rad}$，量测延迟概率 $\lambda=0.3$.

假设机动目标在 1 ~ 100s 以 $\Omega_1=-2°/\mathrm{s}$ 作转弯运动，在 $k=101\mathrm{s}$ 时转弯角速度突变为 $\Omega_2=2°/\mathrm{s}$ 作转弯运动，在 $k=201\mathrm{s}$ 时转弯角速度突变为 $\Omega_3=-2°/\mathrm{s}$ 作转弯运动，并持续到 300s.

本文提出的联合估计与辨识算法和扩维法的初始状态以及协方差分别设置为：

$$\boldsymbol{x}_0^a=[10\mathrm{km}\quad 0.3\mathrm{km/s}\quad 10\mathrm{km}\quad 0.3\mathrm{km/s}\quad -1°/\mathrm{s}]^{\mathrm{T}}$$

$$\boldsymbol{P}_0^a=\mathrm{diag}[100\mathrm{m}^2\quad 10\mathrm{m}^2/\mathrm{s}^2\quad 100\mathrm{m}^2\quad 10\mathrm{m}^2/\mathrm{s}^2\quad 100(\mathrm{mrad/s})]$$

为了公平比较，本文仿真进行了 300 次独立的 Monte Carlo 运行. 为了评估分析本文提出算法的性能，考虑到在实施目标跟踪仿真时一阶泰勒线性化存在巨大的阶段误差以及无迹变换规则存在的数值不稳定问题，本文仿真将不考虑鲁棒 Student's t 扩展滤波器和 RSTUF，即本文算法将只与现有的 RSTCF、现有的 RSTSCF、现有的带一步任意延迟量测的 CKF、现有的带一步任意延迟量测的粒子滤波（PF，粒子数为 5000）、改进的 RSTCF 进行比较分析. 改进的 RSTSCF、改进的 RSTCF 与现有的 RSTCF、现有的 RSTSCF 的自由度参数为，$v_1=v_2=v_3=v=5$.

为了比较上述滤波器的性能，使用状态变量的均方根误差（Root-Mean Square Error，RMSE）和平均均方根误差（Averaged RMSE，ARMSE）作为评价指标. 定义如下：

$$\mathrm{RMSE}(k)=\sqrt{\frac{1}{M}\sum_{i=1}^{M}(\boldsymbol{x}_k^i-\hat{\boldsymbol{x}}_k^i)^2}$$

$$\mathrm{ARMSE}(k)=\sqrt{\frac{1}{MT}\sum_{k=1}^{T}\sum_{i=1}^{M}(\boldsymbol{x}_k^i-\hat{\boldsymbol{x}}_k^i)^2}$$

其中，$\boldsymbol{x}_k^i$ 和 $\hat{\boldsymbol{x}}_k^i$ 分别为第 i 次 Monte Carlo 运行时的第 k 时刻的估计值和真实值. $M=300$ 表示 Monte Carlo 仿真次数.

图 1 ~ 5 分别展示了状态变量不同分量的 RMSE，从图中可以看出，在量测随机延迟条件下，现有的 RSTCF 不可避免地发生了发散现象，而现有的 RSTSCF 尽管比 RSTCF 的整体稳定性较好，但是从图 1 ~ 5 以及表 2 显示的不同滤波器的平均均方根误差可以看出目标状态估计的误差依然很大，无法满足目标跟踪的精度要求. 从图中还可以看出，现有的带一步任意延迟量测的 CKF 在过程噪声与量测噪声均为厚尾噪声等非高斯噪声时，该算法在 x 和 y 方向上的速度估计以及角速度的估计上还能维持一定的收敛性，但是在 x 和 y 方向上的距离估计上则存在着发散现象. 现有的带一步任意延迟量测的 PF 是一种常见的处理量测随机延迟及非高斯噪声背景下的状态估计方法，从图中及表 2 中可以直观地看出，在本次仿真中，粒子数为 5000 的粒子滤波算法整体的估计性能较好，精度较高，性能也比较稳定，但是从表 3 显示的算法运行时间来看粒子滤波算法的运算时间太长，难以满足目标跟踪对于实时性的要求，而基于本文提出的量测随机延迟下带厚尾过程噪声和量测噪声的鲁棒 Student's t 滤波器框架，并分别采用 3 阶球面-相径容积规则、随机 Student's t-球面相径容积规则设计的 3 阶鲁棒 Student's t 容积滤波器和鲁棒 Student's t 随机

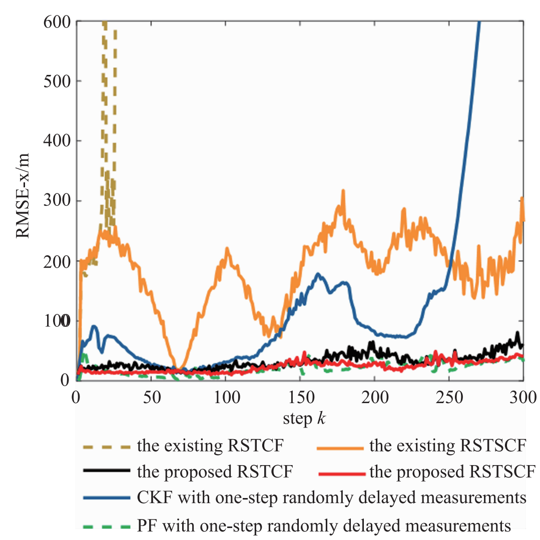

图1　不同滤波器的状态第一分量的RMSE

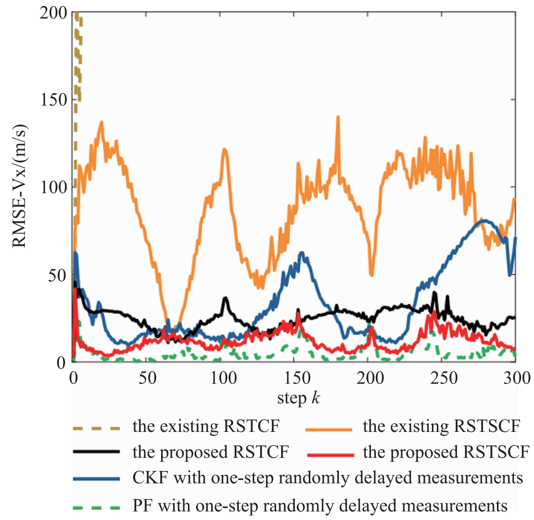

图2　不同滤波器的状态第二分量的RMSE

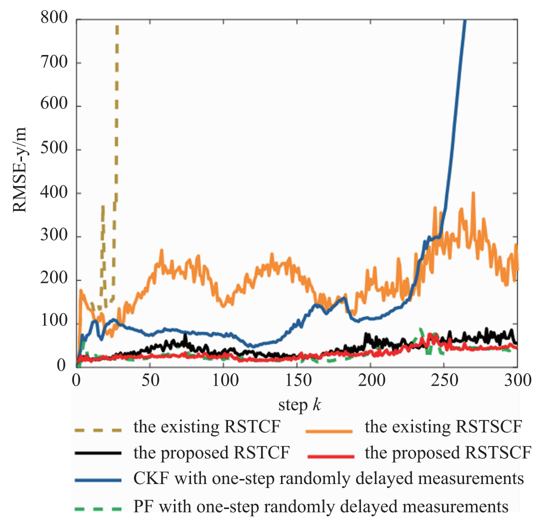

图3　不同滤波器的状态第三分量的RMSE

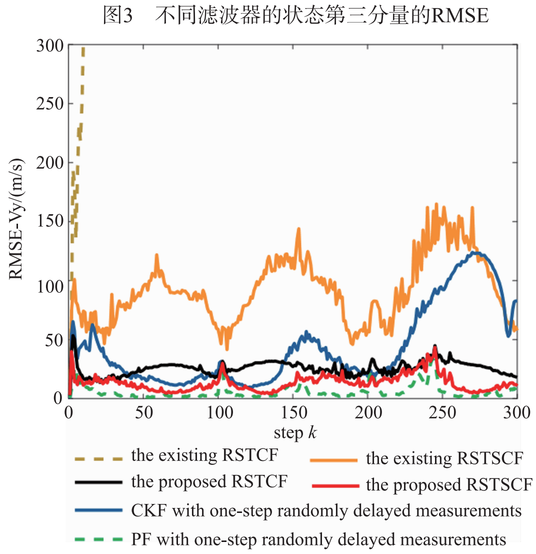

图4　不同滤波器的状态第四分量的RMSE

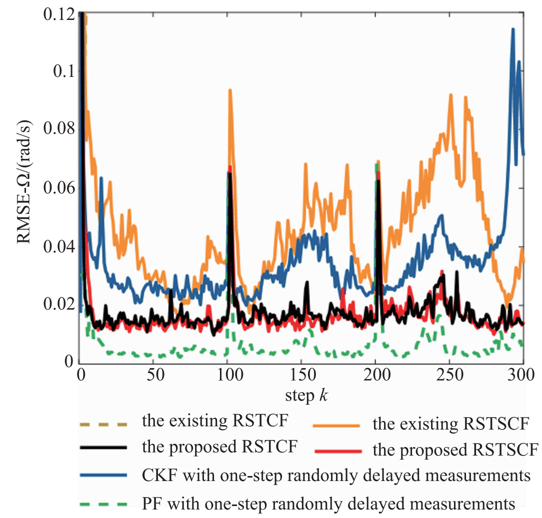

图5　不同滤波器的状态第五分量的RMSE

容积滤波器的整体估计性能较为稳定,从表 1 中可以看出本文提出的算法的 ARMSE 与现有的带一步任意延迟量测的 PF 估计性能与精度相当,但是运算时间成本远远低于现有的粒子滤波算法,性价比较高,从图中还可以看出改进的 RSTSCF 的估计性能要比改进的 3 阶 RSTCF 的估计精度要高,尤其是在 x 和 y 方向上的速度估计上,这一点也可以从表 2 中进一步得到证实.

表 1　不同滤波器的平均均方根误差(ARMSE)对比

滤波器	x(m)	V_x(m/s)	y(m)	V_y(m/s)	Ω(rad/s)
现有的 RSTCF	NaN	NaN	NaN	NaN	NaN
现有的 RSTSCF	192.71	90.89	208.73	95.91	0.0560
带一步延迟量测的 CKF	321.11	38.83	468.63	54.72	0.0384
带一步延迟量测的 PF	23.69	6.87	34.69	8.24	0.0101
改进的 RSTCF	34.68	25.29	46.24	26.12	0.0268
改进的 RSTSCF	25.24	12.93	33.51	14.97	0.0268

为了进一步验证本文所提算法的鲁棒性和优越性,研究了该算法与上述滤波器在不同污染度 p 的情况下的估计性能. 量测延迟概率 $\lambda=0.3$,污染度 $p=0.05, 0.10, \cdots, 0.25, 0.30$.

表 2　不同滤波器的运行时间对比

滤波器	带一步延迟量测的 PF	改进的 RSTCF	改进的 RSTSCF
运行时间(s)	129.39	0.21	7.71

图 6～10 分别展示了本文算法与上述滤波器在污染度 $p=0.05, 0.10, \cdots, 0.25, 0.30$ 时状态变量和转弯角速度的 ARMSE. 从图中看出,粒子滤波算法整体估计的 ARMSE 较低,在位置 ARMSE 上,本文所提算法与粒子滤波算法相当,在速度 ARMSE 与转弯角速度 ARMSE 上,本文所提算法略高于粒子滤波算法,改进的 RSTCF 算法在不同的污染度下仍然能够保持一定的收敛性,而现有的 RSTCF、RSTSCF 以及带一步延迟量测的 CKF 则面临着发散的风险.

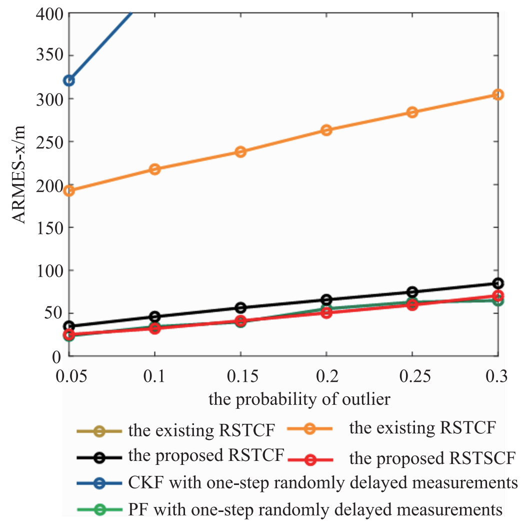

图6　不同滤波器的状态第一分量的ARMSE

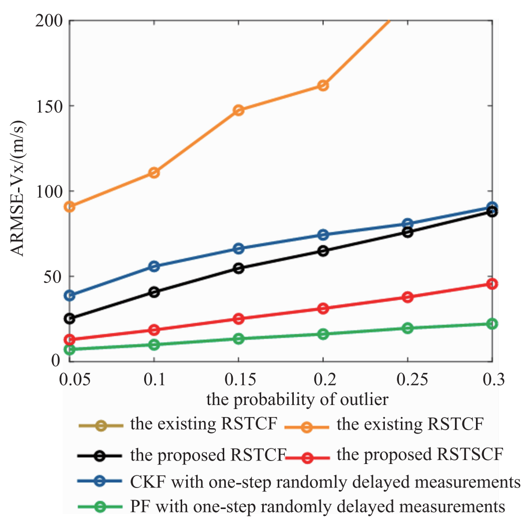

图7　不同滤波器的状态第二分量的ARMSE

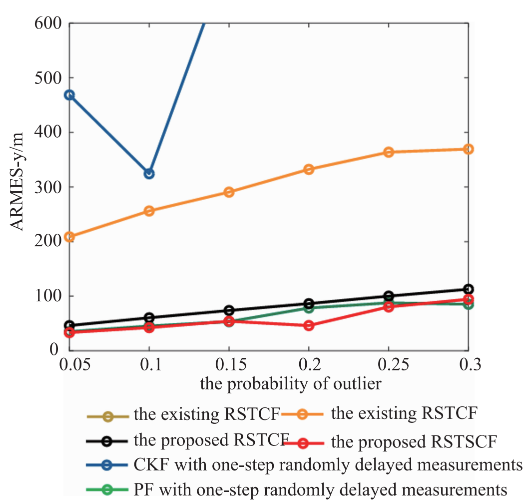

图8　不同滤波器的状态第三分量的ARMSE

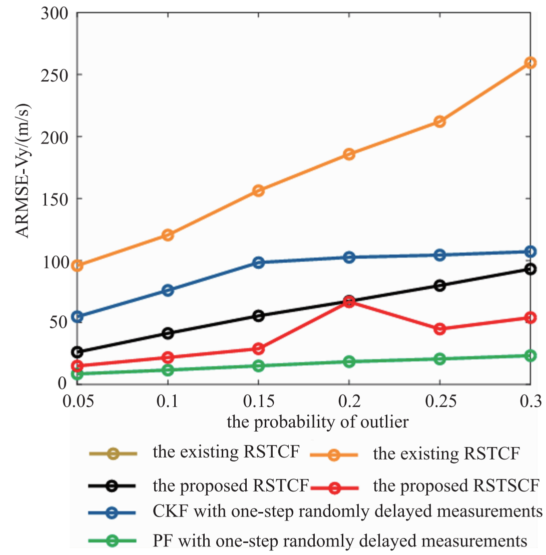

图9　不同滤波器的状态第四分量的ARMSE

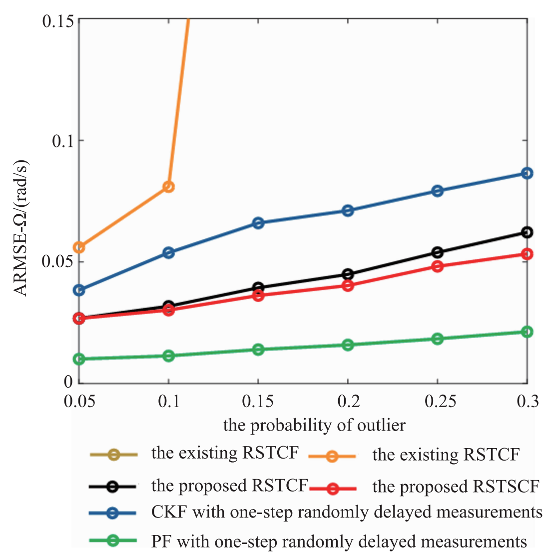

图10　不同滤波器的状态第五分量的ARMSE

6　总结

本文通过充分考虑量测一步随机延迟特性以及过程噪声和量测噪声的厚尾特性,通过采用一组服从 Bernulli 分布的随机序列来描述系统中可能存在的量测一步随机延迟现象,并采用 Student's t 分布来刻画过程噪声和量测噪声中存在的"厚尾"特性,在此基础之上,推导了量测随机延迟条件下带厚尾过程噪声和量测噪声的鲁棒 Student's t 滤波器框架,并基于随机 Student's t－球面相径容积规则进行非线性积分计算,设计了一种鲁棒的 Student's t 随机容积滤波器. 最后通过仿真验证了本文算法比现有算法具有更加稳定的估计性能与更高的估计精度.

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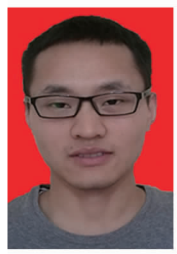

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